

Long lecture

## **The study of envelopes in a CAS environment**

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In considering future developments in Mathematics Education, a graduate course, *Integration of methods in mathematics through a Computer Algebra System (CAS)*, is offered to Israeli Mathematics teachers at the Weizmann Institute of Science. The course is intended to extend their mathematical knowledge and perspectives. For this purpose we present specific topics, rather than whole subjects, that can be studied through a variety of mathematical methods. (Mann and Zehavi, 1998; Zehavi, 2004).

It is our responsibility and challenge as educators to identify pedagogical advantages of the technology for broadening learning opportunities. One way to do so is by introducing topics from the classical mathematics as recommended by Cuoco and Levasseur (2003). Topics that were considered too technical for students become tractable by using the technology to perform the algebraic manipulations and to construct visualization. The illustration of classical concepts and methods in differential geometry using CAS technology provides examples of the new educational possibilities (Migliozzi 1995, Florence 1997).

However, envelopes are not just a beautiful classical topic in differential geometry; it can be studied with various methods from Algebra, Geometry and Calculus. Thom (1962) complained about its disappearance from the curriculum. We claim that the rich variety of methods invite the construction of mathematical relationships.

Using a computerized environment, especially a Computer Algebra System, we may have a revival of a classical topic.

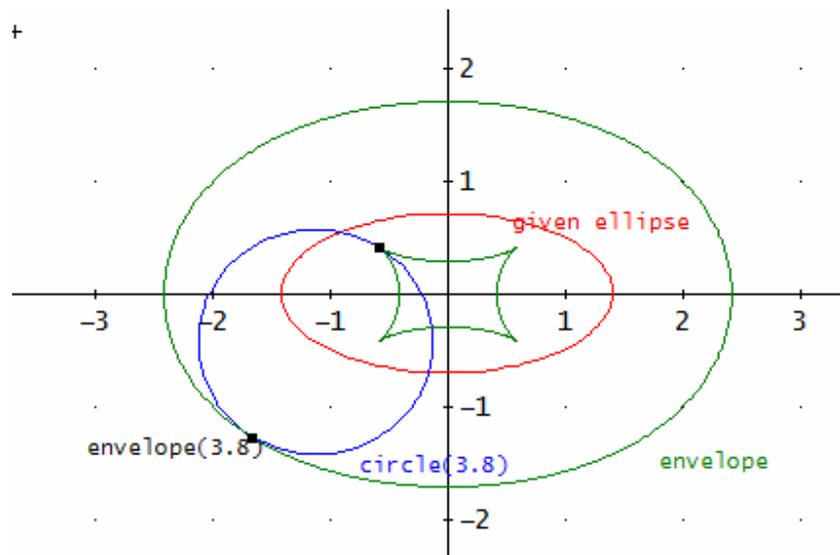
First we show a couple of classical examples, among them:

1. The family of tangents to a given plane curve.
2. The family of curvature circles of a given plane curve.

The CAS based tasks enable the student to develop the needed skills to deal with more complicated situations. In particular, the usage of animations and of slider bar implemented into Derive transforms the study of envelopes from a static study of curves to a dynamical one, by sliding curves and points to build the envelope.

The parallel usage of different registers of representations is multifold. On the one hand, we switch between implicit and parametric representations of curves. On the other hand, using technology, we switch between algebraic, numerical and graphical.

The dynamical features provided by Derive (or other software like Maple, GeoGebra, etc.) makes experimentation worth. The concept of an envelope is made concrete via its CAS realizations. We illustrate this in advanced situations. In particular, we focus on the appearance of envelopes with more than one component and also with singular points.



## References

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R. Thom (1962): Sur la théorie des enveloppes, J. Math. Pures et Appl. 41, 177-192.

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