Short lecture

# Using Fourier series to control mass imperfections in vibratory gyroscopes

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**Abstract**

When a vibrating structure is subjected to a rotation, the vibrating pattern rotates at a rate (called the precession rate) proportional to the inertial angular rate. This is known as Bryan's Effect and it is employed to calibrate the resonator gyroscopes that are used to navigate in outer space, the stratosphere and under the polar cap. Standing waves can exist as stable vibrating patterns in perfect structures, such as a solid annular disc that can be used as a resonator gyroscope. Any manufacturing imperfections in the disc destroy the standing waves. Resonator gyroscopes cannot be manufactured without imperfections (anisotropies) and these imperfections cannot be ignored because they cause departures from ideal mass, stiffness and damping distributions and therefore affect resonator dynamics. Using a CAS in order to illustrate this to undergraduates with a working knowledge of college calculus, we consider the vibrating pattern of a slowly rotating solid annular disc where a slight mass is present. The influence of such a mass imperfection (modelled by a Fourier series) was discussed at the TIME2012 conference. In this paper we introduce a series of capacitors around the disc in order to control the influence of the mass imperfection. A Fourier series is once again introduced to model the influence of the capacitors on the equations of motion of the vibrating disc. This analyses indicates how the capacitors may be arranged in order to neutralize the effects of the mass imperfection. We use the computer algebra system (CAS) Mathematica to do the analysis involved, rendering this work accessible to undergraduate students with a working knowledge of college calculus and basic physics or mechanics (such as senior Engineering Mathematics students). Using a symbolic manipulator such as Mathematica to do the "book-keeping" eliminates the plethora of technical detail that arises during calculations of a highly technical nature, allowing the aforementioned students to focus on the salient parts of the analysis, producing results that might have been beyond their capabilities without the aid of a CAS.

**Keywords**

CAS; Senior undergraduates; Fourier series; Resonator gyroscopes; Capacitor array; Bryan's effect; Rotating disc; Mass imperfection.